Enrollment No: ______________________ Exam Seat No:_____________________

C. U. SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Engineering Mathematics- IV

Subject Code: 4TE04EMT2 **Branch: B.Tech (All)**

Semester: 4 **Date: 19/09/2022** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **[14]**

a) If $f(x)$ is odd function then **(01)**
(a). $B(\lambda)=0$ (b). $A(\lambda)=0$ (c). Both a and b (d). None of these

b) E equal to **(01)**
(a). $1+\Delta$ (b). $1-\Delta$ (c). $\Delta+\nabla$ (d). None of these

c) If $f(z)=u(x,y)+iv(x,y)$ is analytic then $f^{'}(z)=$ ______. **(01)**

(a). $u_x+i\,v_x$ (b). $u_x-i\,v_x$ (c). $u_y+i\,v_x$ (d). $u_x+i\,v_y$

d) The value of $\int_c \frac{dz}{z-5}$. $C:|z|=1$ **(01)**

(a). $2\pi i$ (b). $-2\pi i$ (c). $4\pi i$ (d). 0

e) $curl(grad\phi)=$ _______. **(01)**

(a). $\text{div}\,\vec{f}$ (b). 1 (c). 0 (d). $grad\phi$

f) Which of the following is true if a vector $\vec{f}$ is Irrotational? **(01)**

(a). $\text{curl}\,\vec{f}=\vec{0}$ (b). $\vec{f}=\vec{f}$ (c). $\text{div}\,\vec{f}=0$ (d). None of these

g) In Gauss - Jordan method coefficient matrix reduces into **(01)**
(a). Row Matrix (b). Lower Triangular Matrix
(c). Column Matrix (d). Diagonal Matrix

h) The nth difference of polynomial of degree n is ___ **(01)**
(a). $Constant$ (b). $Zero$ (c). $n!$ (d). None of these

i) According to green's theorem $\oint Pdx+Qdy=$ _____ **(01)**

(a). $\iint_R Pdx + Qdy$ (b). $\iint_R \left(\frac{\partial Q}{\partial y} - \frac{\partial P}{\partial x}\right) dxdy$

(c). $\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy$ (d). None of these

j) Which one of the following method is more rapid in convergence than Gauss-Jacobi method **(01)**
(a). Gauss- elimination method (b). Gauss- Jordan method
(c). Gauss Seidel method (d). None of these

k) Putting $n = 2$ in Newton- cote's formulae, we get ______ **(01)**
(a). Trapezoidal Formula (b). Simpson's $\frac{1}{3}$ rule
(c). Simpson's $\frac{3}{8}$ rule (d). None of these

l) $\delta =$ ____ **(01)**
(a). $\frac{\Delta}{E^{\frac{1}{2}}}$ (c). $E^{\frac{1}{2}} - E^{-\frac{1}{2}}$
(b). $E^{\frac{1}{2}} + E^{-\frac{1}{2}}$ (d). None of these

m) True or false: $u_x = v_y, u_y = -v_x$. **(01)**

n) Write FourierCosine Integral formula. **(01)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **[14]**

a) Given $\vec{u} = xy\hat{\imath} + (2xz - y^2)\hat{\jmath} + yz\hat{k}$ and $v = xy + y^2z + z^2$ then find $\nabla \cdot \vec{u}$, $\nabla \cdot v$ and $\nabla \times \vec{u}$. **(06)**

b) If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, show that $grad r = \hat{r}$ **(04)**

c) Prove that $\vec{f} = \frac{x\hat{\imath} + y\hat{\jmath}}{x^2 + y^2}$ is Solenoidal. **(04)**

Q-3 **Attempt all questions** **[14]**

a) Express $f(x) = \begin{cases} e^{kx} & ; x < 0 \\ e^{-kx} & ; x > 0 \end{cases}$ as a fourier integral and hence show that **(05)**

$$\int_0^\infty \frac{\cos \lambda x}{\lambda^2 + k^2} d\lambda = \frac{\pi}{2k} e^{-kx}; \text{if } x > 0, k > 0$$

b) Find the fourier transform of $e^{-a|x|}, a > 0$ and deduce that **(05)**

$$\int_0^\infty \frac{\cos \lambda x}{\lambda^2 + a^2} d\lambda = \frac{\pi}{2a} e^{-a|x|}$$

c) Find the fourier cosine and sine transforms of the function **(04)**

$$f(x) = \begin{cases} k \text{ if } & 0 < x < a \\ 0 & \text{if } x > a \end{cases}$$

Q-4 **Attempt all questions** **[14]**

a) Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic. Also Find Harmonic conjugate of $u(x, y)$. **(06)**

b) Check the function $f(z) = e^z$ satisfy C-R Equation or not and find $f'(z)$ if possible. **(04)**

c) Determine the Mobius Transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = -5, w_2 = -1, w_3 = 3$ respectively. What are the invariant points of the transformation? **(04)**

Q-5 **Attempt all questions** **[14]**

a) Using Stoke's theorem, find $\oint_C 2xy^2zdx + 2x^2yzdy + (x^2y^2 - 2z)dz$, where C is $x^2 + y^2 + z^2 = a^2$ boundry of hemi sphere. **(06)**

b) Using Green's theorem, evaluate $\oint_C (xy + y^2)dx + dy$ where C is curve formed by $y = x^2$ and $y = x$. **(06)**

c) State Stoke's Theorem. **(02)**

Q-6 **Attempt all questions** **[14]**

a) Given **(05)**
$\sin 45° = 0.7071$,$\sin 50° = 0.7660$,$\sin 55° = 0.8192$, $\sin 60° = 0.8660$ then find $\sin 52°$ using Newton's forward Interpolation formula.

b) Use Lagrange's interpolation formula to find the value of *f(x)* when **(05)**

$$x = 1.$$

x	0	2	3	6
$f(x)$	648	704	729	792

c) If $y_0 = 3, y_1 = 12, y_2 = 81, y_3 = 2000,$ and $y_4 = 100$ then find $\Delta^4 y_0$. Also write Newton's divided difference formula. **(04)**

Q-7 **Attempt all questions** **[14]**

a) Obtain Picard's second approximation solution of the initial value problem **(05)**
$\frac{dy}{dx} = x^2 + y^2$ for $x = 0.4$ correct to four decimal places, given that $y(0) = 0$.

b) Using Taylor series method, find $y(1.1)$ correct to four decimal places, given that **(05)**

$$\frac{dy}{dx} = xy^{\frac{1}{3}}, y(1) = 1.$$

c) Dividing the range into 10 equal parts, find the approximate value of **(04)**

$$\int_0^{\pi} \sin x \, dx \text{ by using simpson's } \frac{1}{3} \text{ rule.}$$

Q-8 **Attempt all questions** **[14]**

a) Solve by using Gauss-Jordan method **(05)**

$$10x + y + z = 12, 2x + 10y + z = 13, x + y + 5z = 7$$

b) Sove the following system by using Gauss-Seidel method **(05)**

$$2x + y + z = 4, x + 2y + z = 4, x + y + 2z = 4$$

c) Show that $\sinh x \cdot \sin y$ is Harmonic function. **(04)**